



# Differential Equations and Dynamical Systems DEDS

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# Conference Proceedings Report

Coordinators: Youssef EL FOUTAYENI & Chaouki AOUITI  
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# Conference Proceedings Report

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## ABOUT DEDS'2022

The Geometry, Analysis, and Applied Mathematics Laboratory (University of Carthage, Faculty of Sciences of Bizerte, Tunisia) AND the Analysis, Modeling, and Simulation laboratory (Hassan 2 University of Casablanca, Faculty of Sciences Ben M'Sik, Morocco) organize the second edition of the international conference on Differential Equations and Dynamical Systems DEDS'2022. We will be inviting our community to engage in new ways, and hope that DEDS 2022 will be an important space for discussion on Differential Equations and Dynamic Systems.

The topics of the conference will be as follow

[1] Population dynamics, control and automation

[2] Recurrent neural networks

## FOREWORD & ACKNOWLEDGEMENTS

The second International Conference on Differential Equations and Dynamical Systems DEDS'2022 is aimed to bring researchers and professionals to discuss recent developments in both applied mathematics and computer science and to create a professional knowledge exchange platform between mathematicians, computer science and other disciplines. This conference is the result of international cooperation bringing together African and European universities. It is a privileged place for meetings and exchanges between young researchers and high-level African and international decision makers in the fields of mathematics and applied computing.

This conference has several major objectives, in particular:

- To bring together doctoral students and research professors in the fields of applied sciences and new technologies.
- To consolidate the scientific cooperation between the university and the socio-economic environment in the field of applied sciences.
- To allow young researchers to present and discuss their research work before a panel of specialists and university professors.
- To contribute to the development of a database, which can help decision makers to opt for a better management strategy.

The abstracts of these conference proceedings were presented at the second International Conference on Differential Equations and Dynamical Systems DEDS'2022. These conference proceedings include abstracts that underwent a rigorous review by two or more reviewers. These abstracts represent current important work in the field of Mathematics and are elaborations of the DEDS conference reports.

We wish to acknowledge the conference program committee and reviewers, for their substantial contributions and our institutions, for their support.

Sincerely,

*On behalf of Organizing Committee of DEDS 2022*

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*Prof. Youssef EL FOUTAYENI  
Hassan II University of Casablanca  
Morocco*

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*Prof. Chaouki AOUITI  
University of Carthage  
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Prof. Khalid HATTAF  
CRMEF of Casablanca | Morocco



Prof. Khadija NIRI  
Hassan II University | Morocco

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## Modeling and mathematical analysis of an immunological viral infection model

Mly Ismail ELKARIMI<sup>1</sup>, Khalid HATTAF<sup>1,2</sup>, Noura YOUSFI<sup>1</sup>

### Abstract

Several viruses, such as human immunodeficiency virus (HIV), hepatitis B virus (HBV), hepatitis C virus (HCV), and more recently severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), which is responsible for the coronavirus disease 2019 (COVID-19), continue to cause new infected cases, numerous health problems, deaths and socio-economic damage worldwide.

Currently, many mathematical models have been proposed and developed in order to understand the dynamics of these viruses. Therefore, the main aim of this work is to propose and analyze the dynamics of an immunological viral infection model. We first investigate the existence of equilibria and the well-posedness of the proposed model including nonnegativity and boundedness of the solutions. Also, we establish the local and global stability of equilibria according to the values of the basic reproduction number  $R_0$  and other parameters related to the model. The proposed model includes both modes of transmission and cure of infected cells. Numerical simulation makes it possible to visualize the evolution of the model's variables over time, according to its parameters. These numerical simulations are made with Python in order to support our analytical findings. This work is in line with current research, presenting very useful models in biology, particularly in viral immunology. It generalizes recently published models and opens the way to other research questions.

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1\_ Laboratory of Analysis, Modeling and Simulation (LAMS), Faculty of Sciences Ben M'Sick, Department of Mathematics and Computer Science, Hassan II University of Casablanca, Morocco.

2\_ Equipe de Recherche en Modélisation et Enseignement des Mathématiques (ERMEM), Centre Régional des Métiers de l'Éducation et de la Formation (CRMEF), Casablanca, Morocco.

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## **Modification de la matrice de comparaison incohérente dans le processus hiérarchique analytique "AHP" BADR SANAA, TAHRI MERYEM, YOUSFI NOURA**

### **Résumé**

De nombreuses techniques sont employées pour faciliter la sélection d'une alternative en fonction d'un ensemble de préférences, de critères et d'autres facteurs. La méthode AHP (Analytic Hierarchy Process) conçue par Saaty [1] est la méthode la plus populaire et la plus utilisée. L'AHP effectue des comparaisons par paires pour évaluer la pertinence relative de ces composants (critères, sous-critères, alternatives, etc.) et établit ainsi des priorités [2]. Cependant, avant d'utiliser une matrice de comparaison par paire, il faut en vérifier la cohérence, car les gens sont plus susceptibles d'être incohérents que d'être cohérents lorsqu'ils portent des jugements, du fait qu'ils ne peuvent pas estimer avec précision les valeurs de mesure, même à partir d'une échelle connue [3].

Une étape critique de l'AHP consiste à valider la cohérence des jugements en calculant un ratio de cohérence noté CR, qui doit être inférieur à 0,1 [4]. Le décideur devra apporter des modifications si une matrice par paire ne répond pas à ce critère.

Notre objectif principal est d'aider les décideurs à réviser leurs jugements afin de créer des matrices de comparaison cohérentes en utilisant des méthodes permettant de déterminer la valeur qui suscite une plus grande incohérence et en suggérant une nouvelle valeur pour la remplacer.

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## Solving Job-Shop Scheduling Problem by recurrent neural networks and a new hybrid matrix metaheuristic: a comparative study

A. Lotfi Nohair, B. Abderrahim Eladraoui, C. Abdelwahed Namir

### Abstract

Based on the research of Zhang [1] and Willems [2], this communication proposes a recurrent neural network to solve Job-shop scheduling problems [3].

Firstly, the problem was translated in an integer linear programming model which the objective is to minimize the makespan, subject to three types of constraints:

- 1) Starting time constraints (ST units): the starting time of each operation must be a positive integer number.
- 2) Sequence constraints (SC units): An operation can only be scheduled after the preceding ones have ended.
- 3) Resource constraints (RC units): Machine can process at most one job at a time

This integer linear representation has been translated to Hopfield neural network. The proposed network used two parts: main part and feedback part. The main part include neurons representing the starting time of corresponding operation. In the feedback part, we use the network structure for constraint violation. The feedback part consists of three layers: the first layer representing the ST units, the second layer representing SC and the third layer representing RC units. The network was implemented using MATLAB program. Our simulation results was to test the network with both small-size and big-size problems. The quality of results obtained, depends on the initialization of the network. Therefore, we used simulated annealing as local search to improve the performance of the proposed network. The goal is to minimize the energy which includes the makespan and the energy that represents the constraint violation.

Finally, we suggest a comparative study between the recurrent neural network and the new hybrid matrix met heuristic proposed in our article [4].

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## On the asymptotic behavior of some functional differential equations

E.Ait dads, S.Fatajou, and Z.Zizi

### Abstract.

The study of almost periodic solution to functional differential equations has attracted the attention of many researchers, thanks to their applications in various fields of science and engineering. On the other hand, W. F. Eberlein [1] introduced the concept of Eberlein-weakly almost periodic functions as a generalization of almost periodicity in the sense of Bochner. This concept was then deeply investigated by W. M. Ruess, W. H. Summers [2, 3, 4] and applied to study the asymptotic behaviors of various differential equations [5, 6, 7]. Later, Ait Dads, Fatajou and N'Guérékata [8] considered the concept of Stepanov Eberlein-weakly almost periodic function which is a generalization of the classical notion of weak almost periodicity in the sense of Eberlein and showed many fundamental properties about this class. In this work, we provide sufficient conditions ensuring the existence and uniqueness of an Eberlein-weakly almost periodic solution for some functional differential equations in Banach space when the forcing term is only Eberlein-weakly almost periodic in a weaker sense, namely, the Stepanov sense. Then, we treat the almost periodic case. Our results continue the investigations done in the literature about the existence of almost periodic and almost automorphic solutions. The theoretical results are then applied to some model occurring from physics.

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## Fractional Kpz system involving a non-local gradient term S. BOUKARABILA

### Abstract

Our work deals with the nonlocal version of Hamilton-Jacobi equation with nonlocal gradient term.

The main considered problem is the following system:

$$\begin{cases} u_t + (-\Delta)^s u &= |(-\Delta)^{\frac{s}{p}} v|^p + f(x, t) & \text{in } \Omega_T \\ v_t + (-\Delta)^s v &= |(-\Delta)^{\frac{s}{q}} u|^q + g(x, t) & \text{in } \Omega_T \\ u(x, 0) = v(x, 0) &= 0 & \text{in } \Omega \\ u(x, t) = v(x, t) &= 0 & \text{in } (\mathbb{R}^N \setminus \Omega) \times (0, T) \end{cases} \quad (1)$$

Where  $\Omega$  is a bounded domain in  $\mathbb{R}^N$ ,  $p, q \geq 1$  and  $f, g$  are nonnegative data.

By  $(-\Delta)^s$  we denote the fractional Laplacian given with

$$(-\Delta)^s u(x) := a_{N,s} \text{ P.V. } \int_{\mathbb{R}^N} \frac{u(x) - u(y)}{|x - y|^{N+2s}} dy, \quad s \in (0, 1),$$

Our aim is to show under which conditions on the given data and the exponent  $p, q$  we get the existence of a nonnegative weak solution for the system (1). In some particular cases, we are able to prove that these conditions are optimal.

These results are part of the paper [1].

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## A virotherapy model with the new generalized Hattaf fractional derivative

Majda EL YOUNOUSSI<sup>1</sup>, Khalid HATTAF<sup>1,2</sup>, Noura YOUSFI<sup>1</sup>

### Abstract

Lastly, virotherapy takes the attention of many researchers. This therapy is one of the promising cancer treatments. Its big advantage is to kill cancer cells without harming normal cells. To develop the oncolytic viral therapy, and forecast their effectiveness and potential outcomes, we use mathematical models.

Some of these models are based on fractional derivatives. Because of its power to model the biological interactions with a respect of the present and the past time, fractional derivative enables to model the effect of memory and the hereditary properties of biological phenomena.

In this work, a new fractional model for virotherapy using the generalized Hattaf fractional (GHF) derivative with non-singular kernel is proposed. The GHF derivative used in this model is a generalization of the Caputo-Fabrizio fractional derivative, the Atangana-Baleanu fractional derivative, and the weighted Atangana-Baleanu fractional derivative. Furthermore, we study the existence and uniqueness of the solution of the proposed model. Moreover, we investigate the stability analysis with the GHF derivative of the model's equilibria.

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## Optimal control of a frictional contact problem

Aziza. Bachmar<sup>1</sup>

### Abstract

We consider a mathematical model which describes a contact between a nonlinear elastic body and a foundation[1]. The contact is frictional and modelled with normal compliance associated to unilateral constraint. The goal of this paper is to study an optimal control problem which consists of leading the stress tensor as close as possible to a given target, by acting with a control on the boundary of the body. We derive a variational formulation of the mechanical problem and prove its unique weak solvability. Then, we introduce a penalized contact problem which we prove existence, uniqueness and convergence results. We state an optimal control problem for which we prove the existence of solution[2],[3]. Finally, we consider an optimal control problem associated to the penalized contact problem and prove a convergence result.

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## Customer Segmentation Using the k-Means Clustering Algorithm

Fayçal Messaoudi, Manal Loukili, Mohammed El Ghazi

**Abstract** Au fur et à mesure que la technologie progresse et que de plus en plus d'entreprises apparaissent chaque jour, il est devenu essentiel pour toutes les entreprises d'appliquer des stratégies de marketing pour rester sur le marché, car la concurrence est désormais féroce. La base de clients s'élargissant de jour en jour, il est devenu difficile pour les entreprises de répondre aux besoins de chaque client, raison pour laquelle les technologies d'intelligence artificielle jouent un rôle majeur dans le démêlage des modèles cachés stockés dans la base de données de l'entreprise. La segmentation de la clientèle permet aux entreprises de fixer des prix appropriés pour les produits, d'élaborer des campagnes de marketing personnalisées, de concevoir une stratégie de distribution optimale, de sélectionner des caractéristiques de produits spécifiques à développer et de donner la priorité au développement de nouveaux produits, ce qui leur permet de mieux servir leurs clients et donc de les satisfaire. La segmentation est le processus qui consiste à diviser une population d'individus en sous-ensembles homogènes qui partagent un ou plusieurs attributs communs. Chaque sous-ensemble constitue un segment. La segmentation est donc la condition de possibilité du ciblage. Le clustering s'est avéré être un moyen efficace pour effectuer une segmentation de la clientèle. Il s'agit d'une technique d'apprentissage automatique qui fait partie de l'apprentissage non supervisé et qui a donc la capacité de trouver des clusters sur des données non étiquetées. Il existe plusieurs algorithmes de clustering, le clustering K-means étant le plus approprié pour le clustering de l'ensemble des données. La fidélisation des consommateurs et la durée d'attention sont des problèmes majeurs auxquels le secteur des affaires est confronté aujourd'hui. Avec l'aide des algorithmes K-means, les entreprises peuvent facilement identifier le groupe auquel appartient chaque client, ce qui facilite la gestion de la relation client. L'objectif de cet article est d'appliquer l'algorithme de clustering K-means à notre ensemble de données, afin de subdiviser les clients en différents groupes en fonction de leurs données personnelles telles que l'âge, le sexe, le revenu annuel et le score de dépenses. Notre étude comprend les étapes suivantes : la collecte des données, puis le prétraitement des données, puis l'analyse des données à l'aide de la méthode de clustering K-means, suivie du choix du nombre de clusters à l'aide de la méthode du coude, et enfin l'analyse des résultats.

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# DEDS 2022

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## New criteria on time scale for neutral dynamic equations with unbounded delay in terms of Stability

I. Daira<sup>1</sup>, A.Ardjouni<sup>2</sup>

### Abstract

The study of dynamic equations brings together the traditional research areas of (ordinary and partial) differential and difference equations. It allows one to handle these two research areas at the same time, hence shedding light on the reasons for their seeming discrepancies. In fact, many new results for the continuous and discrete cases have been obtained by studying the more general time scales case ; We have studied dynamic nonlinear equations with functional delay on a time scale and have obtained some interesting results concerning the existence of periodic solutions (see [1]-[3]) and this work is a continuation. Here, we focus on two neutral nonlinear dynamic equations which, for our delight, have not been yet studied by mean of fixed point technical on time scales. There is no doubt that the Liapunov methods have been used successfully to investigate stability properties of wide variety of ordinary, functional and partial equations. Nevertheless, the application of this method to problem of stability in differential equations with delay has encountered serious difficulties if the delay is unbounded or if the equation has unbounded term. It has been noticed that some of theses difficulties vanish by using the fixed point technic. Other advantages of fixed point theory over Liapunov's method are that the conditions of the former are average while those of the latter are point wise.

we consider the neutral nonlinear dynamic equations with unbounded delay , In this work we use the fixed point technique based on the contraction mapping theorem to prove that the zero solution is stable and illustrate our theory by giving example

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# DEDS 2022

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## Deep Learning in IoT Object Identification Systems: A Review

L. ELHALOUI<sup>1</sup>, S. EL FILALI<sup>2</sup>, H. BENLAHMER<sup>3</sup>

<sup>1, 2, 3</sup>, LTIM, Hassan II University of Casablanca, Morocco

### Abstract

In recent years, there has been a huge growth in connected objects in our environment, users are buying more and more connected objects in order to benefit from new services, to improve their quality of life.

The main objective of this work was to propose a system for identifying connected objects in an IoT environment using the characteristics of the network flow.

The operation of this connected object identification system in an intelligent environment goes through three stages, firstly requires a network flow analysis stage to build our base of characteristics that can be used by deep learning algorithms. Next, a step of extracting parameters characterizing the different classes is carried out from IoT traffic traces, as well as a step of classifying all the parameters extracted to obtain the identity of the object considered using one or more models such as RNN, RBFN, MLP and auto-encoders.

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## Controllability of the 1-d wave equation in some intervals with variable ends

Seyf Eddine Ghenimi, Abdelmouhcene Sengouga

### Abstract

In this lecture we will consider the one-dimensional wave equation in an interval with two linearly variable ends with a constant speed  $v$ . We assume that  $v$  is strictly less than the speed of propagation of the wave, i.e.

$$0 < v < 1.$$

Then, we define the following interval with variable ends

$$\Omega_t := (vt, L + vt), \quad t \geq 0.$$

Let us now consider the wave equation, with homogeneous Dirichlet boundary conditions,

$$\begin{cases} \psi_{tt} - \psi_{xx} = 0, & \text{for } x \in \Omega_t, \quad t \geq 0, \\ \psi(vt, t) = \psi(L + vt, t) = 0, & \text{for } t \geq 0, \\ \psi(x, 0) = \psi^0(x), \quad \psi_t(x, 0) = \psi^1(x), & \text{for } x \in \Omega_0, \end{cases}$$

where  $\psi(x, t)$  is the transverse displacement of the string and the subscripts  $t$  and  $x$  stand for the derivatives with respect to time and space, respectively.

We establish the existence and uniqueness of the solution by a series formula, then we show that the energy of the solution is bounded and periodic in time with period  $T_v = 2L/1 - v^2$ . We also show exact boundary observability at each of the ends of the interval. Moreover, using the Hilbert Uniqueness Method (HUM), we deduce the exact boundary controllability results.

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# DEDS 2022

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## Finite-time passivity of neutral-type Hopfield neural networks with time-varying delay via Wirtinger-type inequality

*Chaouki Aouiti, Farid Touati*

### Abstract

During the last years, a lot of attention has been concentrated on the study of neural networks because they are a class of important mathematical models. Artificial neural networks are computer systems inspired by biological neural networks that form the brains of animals. In the past few decades, these systems gradually improve and branch out [1]. One of them is Hopfield neural networks which had been proposed by John Joseph Hopfield in 1982 in order to solve some complicated problems in many fields like signal processing, pattern recognition, combinatorial optimization and so on.

The theory of passivity is an important concept of the automatic for the analysis and for the control of systems whose certain input/output characteristics are established in terms of energetic criteria. The notions of passivity are adapted to several scientific fields and are effective for the regulation of electrical, mechanical and electromechanical systems present in several fields of engineering, such as robotics, power electronics, aeronautics and so on. The basic premise of passivity theory is that a system's passive characteristics may make it internally stable. Since the 1970s, the control community has paid close attention to passivity theory, which is closely connected to circuit analysis. The passivity theory is a strong tool for assessing system stability (see [2]) and has applications in many fields, including complexity (see [3]), chaotic control and synchronization (see [4]) and signal processing (see [5]).

In this presentation, we are interested in the study of the finite-time passivity of neutral-type Hopfield neural networks with time-varying delay via Wirtinger-type inequality.

We constructed a new Lyapunov–Krasovskii function with triple, four and five integral terms and then utilizing Wirtinger-type inequality technique, we obtained a set of sufficient conditions to prove the finite-time boundedness and finite-time passivity for the addressed model. Lastly, two numerical examples with simulations are given to illustrate the effectiveness of our main results. In this work, we aim to establish the finite-time passivity of the following delayed system:

$$\begin{cases} \dot{x}(t) = -Ax(t) + Bf(x(t)) + Cf(x(t - \tau(t))) + D\dot{x}(t - h(t)) + \omega(t), \\ z(t) = K_1x(t) + K_2f(x(t)) \end{cases}$$

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# DEDS 2022

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## Fixed-Time synchronization of delayed inertial Cohen-Grossberg neural networks

El Abed Assali, Chaouki Aouiti

*University of Carthage, Faculty of Sciences of Bizerta, Department of Mathematics, GAMA  
Laboratory LR21ES10, BP W 7021 Zarzouna, Bizerta, Tunisia.*

### Abstract

In recent years, much attention has been devoted to the studies of inertial Cohen-Grossberg neural networks (ICGNNs) due to their important application in various areas such as signal financial industry, image processing, pattern recognition, control, optimization problems [1,2]. This paper is devoted to studying the fixed-time synchronization of the following inertial Cohen-Grossberg neural networks with time varying delays [3,4]:

$$\frac{d^2x_i(t)}{dt^2} = -\beta_i \frac{dx_i(t)}{dt} - \alpha_i(x_i(t)) \left[ h_i(x_i(t)) - \sum_{j=1}^n a_{ij}(t) f_j(x_j(t)) - \sum_{j=1}^n b_{ij}(t) f_j(x_j(t - \tau_j(t))) + I_i(t) \right],$$

where  $n \geq 2$ ,  $t \geq t_0$ ,  $i = 1, 2, \dots, n$ , where the second derivative is called an inertial term of system;  $x_i(\cdot)$  is the states variable of the  $i$ th neuron at the time;  $\beta_i > 0$  are constants;  $\alpha_i(\cdot)$  denotes an amplification function;  $h_i(\cdot)$  denotes the behaved function;  $a_{ij}(\cdot)$  are the connection weights of the neural networks;  $f_j(x_j(t))$  denotes the activation function of  $j$ th neuron at time  $t$ ;  $\tau_j(t)$  is the time delay of  $j$ th neuron and corresponds to finite speed of axonal signal transmission at time  $t$ ;  $I_i(\cdot)$  the input from outside of the networks. First, by constructing a proper variable substitution, the original can be rewritten as first-order differential system. Second, by utilizing feedback controllers and constructing suitable Lyapunov functionals, several new sufficient conditions guaranteeing the fixed-time synchronization of ICGNNs with time varying delays are obtained based on different finite-time synchronization analysis techniques. The obtained sufficient conditions are simple and easy to verify. Numerical simulations are given to illustrate the effectiveness of the theoretical results.

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## Convolutional Neural Networks Architectures using in imagery

A. Meryem Ameer<sup>1</sup>, B. Najlae Idrissi<sup>2</sup>, C. Cherki Daoui<sup>3</sup>

### Abstract

Processing image has a big place in many domains like medicine, astronomy...it is a set of mathematics operations; segmentation, filtering, classification, these methods are based on mathematics methods and calculus. Image processing has many roles like treat the image to obtain the good results using filtering techniques, localize same place in images using segmentation and detection techniques, classify the images in the suitable class, and help to take decision. In this work, we present some Convolutional Neural Networks Architectures using in image segmentation, we use two architectures one called by SegNet and another named U-Net, each architecture has own principle to segment images basing on number of layers. In addition, we use the Hidden Markov Model: Hidden Markov Chain. These methods are used to segment color images; we compare the three models SegNet, U-Net and Hidden Markov Chain in term of quality and execution time. The obtained results, demonstrate that: SegNet, U-Net and Hidden Markov Chain provide the same results quality. But, Convolutional Neural Networks segments the images fast than Hidden Markov Chain. In generally, the three models are used to segment images by the same principle is the principle of semantic segmentation basing on clustering.

### Keywords

CNN, Semantic Segmentation, SegNet, U-Net, HMC, DICE.

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## Existence of solution for hybrid fractional system with impulsive condition

S. Zerbib, K. Hilal, A. Kajouni

### Abstract

The differential equations involving fractional derivatives in time, compared with those of integer order in time, are more realistic to describe many phenomena in nature (for instance, to describe the memory and hereditary properties of various materials and processes).

Quadratic perturbations of nonlinear differential equations have attracted the attention of several authors. Sometimes a differential equation representing a certain dynamical system is not easily solvable or analyzed, however, the perturbation of such problem in someone manner makes it possible to study the problem with available methods for different aspects of the solutions. Differential equations perturbed in this way are called hybrid differential equations. These equations are interesting equations that form another step for solving problems in modeling field.

Impulsive differential equations are used to describe the evolutionary processes which abruptly change state at some point. This subject has received great importance and remarkable attention from researchers due to its rich theory and its applicability in various branches of science and technology.

In this work, we study the existence of solution for fractional hybrid differential equation involving  $\psi$ -Hilfer fractional derivative with impulsive condition. An existence theorem for fractional hybrid differential equations is proved under the classical technique of Dhage fixed point theorem.

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## New and improved conditions for qualitative behaviors of differential systems with time- varying delay

Cemil Tunç and Osman Tunç

### Abstract

This paper is concerned with some qualitative analyses of solutions of continuous-time systems of delay differential equations (DDEs) of first order. In this paper, certain the continuous-time delay unperturbed and perturbed systems of DDEs are considered, correspondingly. The qualitative analyses solutions of them are connected based on the LKF approach. Firstly, an LKF is defined, and then by aid of this LKF, three novel theorems, Theorems 1-3, which have improved sufficient conditions for asymptotic stability (AS), integrability and bounded solutions, are proved. Two numerical examples are also provided, which illustrate the efficiency of the LKF approach. The given novel results are also more general than that obtained in the past literature.

**Keywords.** System, DDEs, stability, integrability, boundedness, LKF approach, time-varying delay

**Subject Classification:** 34D05, 34K20, 45J05

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UNIQUENESS OF SOLUTIONS FOR NONLINEAR IMPLICIT HADAMARD FRACTIONAL  
DIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS IN A WEIGHTED BANACH SPACE.

**Chahra Kechar, Abdelouaheb Ardjouni**

## Abstract

Fractional differential equations arise from a variety of applications including invarious fields of science and engineering.

In particular, problems concerning qualitative analysis of fractional equations have received the attention of many authors.

Fractional differential equations involving Riemann-Liouville and Caputo fractional derivatives have been studied extensively by several researchers. However, the literature on Hadamard differential equations is not yet as enriched. The fractional derivative due to Hadamard differs from the aforementioned derivatives in the sense that the kernel of the integral in the definition of Hadamard fractional derivative contains alogarithmic function of arbitrary exponent.

The aim of this paper is to prove the existence and uniqueness of solutions for a nonlinear implicit Hadamard fractional differential equation with nonlocal conditions in a weighted Banach space.

Our results are based on the Banach and Krasnoselskii fixed point theorems. An example is given to illustrate our obtained results.

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## Global exponential stability of $(\mu, \nu)$ -pseudo almost automorphic solution of fuzzy Cohen-Grosberg neural networks with time-varying delays

Hediene Jallouli, Chaouki Aouiti

### Abstract

The fuzzy Cohen-Grosberg differential equations with mixed delays and time-varying coefficient will be studied. Then, many interesting results on the functional space of such functions like composition theorems are establish. The theory of this work generalizes the classical results on weighted pseudo almost automorphic functions. Compared with exists works, the activation functions using in this paper are of two variables ( $t$  and  $x(t)$ ) and Lipschitz constants are variable in space  $L^p(\mathbb{R}, d\mu) \cap L^p(\mathbb{R}, dx)$ . By using the fixed-point theorem and some properties of the doubly measure pseudo almost automorphic functions, a set of sufficient criteria are established to ensure the existence, uniqueness and global exponential stability of  $(\mu, \nu)$ -pseudo almost automorphic solution. Finally, a numerical example is provided to illustrate the validity of the proposed theoretical results.

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## The limit cycles for some families of discontinuous piecewise differential systems separated by a non-regular line

L. Baymout, R. Benterki

### Abstract

During these last decades to study discontinuous piecewise differential systems becomes an interesting subject of research due to the important applications of this kind of systems to model natural phenomena. In the qualitative theory of differential equations one of the interesting problems is the detection of the number of limit cycles and their configurations which is remains open till now, we recall that a limit cycle of a planar differential system is an isolated periodic orbit in the set of all periodic orbits of this system. The importance of the existence and the possible configurations of limit cycles comes from the main role of limit cycles in understanding and explaining the dynamics of a given differential system. For discontinuous piecewise differential systems we can find two types of limit cycles, the crossing and the sliding ones, here we are interested in the crossing limit cycles. In this paper the main tool used to prove the existence and the maximum number of crossing limit cycles for some families of piecewise discontinuous differential systems separated by a non-regular line and formed by two differential systems is based on the first integrals of the systems that forms the piecewise discontinuous differential systems. Here we study the case where we have a linear center and one of the six families of isochronous centers. For all these classes, we prove that the maximum number of limit cycles is reached. In summary, We have solved the extension of the second part of Hilbert's 16th problem for this class of piecewise differential systems.

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## Contrôle en mode de glissement à temps fixe pour les réseaux de neurones récurrents discontinus non identiques Chaouki Aouiti, Maissa Bessifi

### Résumé

Cet article étudie le problème très connu dans le domaine des réseaux de neurones qui est la synchronisation en temps fixe des réseaux de neurones récurrents retardés (RNNs). De plus, on suppose que les paramètres des RNNs ne sont pas identiques et que les fonctions d'activation sont discontinues. Pour résoudre le problème de synchronisation en temps fixe, une approche améliorée de contrôle en mode coulissant (SMC) est présentée. Pour la première fois, en appliquant le concept de réponse d'entraînement et l'erreur de synchronisation entre le modèle de réponse d'entraînement, deux nouvelles surfaces de mode de glissement intégrées sont construites de sorte que l'erreur de synchronisation peut converger vers zéro en temps fixe. Pour la deuxième fois, un SMC est créé pour surmonter le problème de synchronisation de temps fixe des RNNs retardés. Sur la base de la théorie de stabilité de Lyapunov, plusieurs critères suffisants sont obtenus pour les RNNs non identiques discontinus considérés avec des modèles de délais variables pour atteindre la synchronisation temporelle fixe. En fin de compte, trois simulations numériques sont données pour illustrer l'efficacité des critères de synchronisation (voir [1], [2]).

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## Analysis Mathematical and optimal control of an Obesity Model

R. Bouajaji, H. Laarabi, A. Abta, M. Rachik, Y. El Foutayeni

### Abstract

Overweight and obesity have been a major health problem in the world. Social contagion is an important factor in the development of obesity, and its identification and control may lead to effective planning in intervening in the obesity epidemic. In this paper, we propose a model that approaches obesity from a mathematical point of view, through epidemiological models, to describe the spread of the obesity as a contagious disease. The study of stability analysis theory we are leading is meant to find the equilibriums for the model. We also studied the sensitivity analysis of the parameters which have an important impact on the basic reproduction number  $R_0$ . Next in order to achieve control of the disease, we consider a control problem relative to the obesity model. We analysed two optimal control strategies for our continuous-time obesity model, our main objective is to predict how the health system can reduce the increasing number of an overweight and obesity as quickly as possible with minimal cost. In the first strategy, the first control characterizes the effectiveness of awareness raising in different media (encouraging people to do physical activities and to follow a healthy diet ...), and where the problem of optimal control is actually a matter of an optimization criterion represented by the minimization of an objective function, aimed at minimizing the number of overweight and then, of course, increasing the number of people at normal weight. In the second strategy, we also introduced the control of medical treatment of obese individuals. Numerical simulation associated with these two strategies has shown the effectiveness in reducing the number of overweight and obesity.

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## A generalized business cycle model with a variable depreciation rate

Sara LASFAR<sup>1</sup>, El Mehdi WARRAK<sup>1</sup>, Khalid HATTAF<sup>1,2</sup>, Noura YOUSFI<sup>1</sup>

### Abstract

The business cycle model is a very important area of research in economics. The appearance of business cycle can be obtained from the nonlinear properties of the economic system and the existence of time delay in the investment process. Some of these models are based on delays, the first one concerns the delay between the decision of investment and its fulfilment. Whereas, the second one concerns the time lag for the investment to be made. On the other hand, many present researches models consider that depreciation rate constant. However, economists believe that people get experience while they are working, which leads to improved work efficiency.

Based on the above mathematical and economic considerations, this work develops analyses the dynamics of a generalized business cycle model with general investment and variable depreciation rate of capital stock. Firstly, we study the existence and the uniqueness of the solution of the developed model. Secondly, the existence, uniqueness and stability of the economic equilibrium are rigorously investigated. Thirdly, the existence of Hopf bifurcation is carefully set up. Finally, numerical simulations are presented to illustrate our analytical results.

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## Optimal control problem for an age-structured model of COVID-19 transmission with fractional derivative

### Abstract

The aim of this study is to model the transmission of COVID-19 and investigate the impact of some control strategies on its spread. We propose an extension of the classical SEIR model, which takes into account the age structure and uses fractional-order derivatives to have a more realistic model. For each age group  $j$  the population is divided into seven classes namely susceptible  $S^j$ , exposed  $E^j$ , infected with high risk  $I_h^j$ , infected with low risk  $I_l^j$ , hospitalized  $H^j$ , recovered with and without psychological complications  $R_1^j$  and  $R_2^j$ , respectively. In our model, we incorporate three control variables which represent: awareness campaigns, diagnosis and psychological follow-up. The purpose of our control strategies is protecting susceptible individuals from being infected, minimizing the number of infected individuals with high and low risk within a given age group  $j$ , as well as reducing the number of recovered individuals with psychological complications. Pontryagin's maximum principle is used to characterize the optimal controls and the optimality system is solved by an iterative method. Numerical simulations performed using Matlab, are provided to show the effectiveness of three control strategies and the effect of the order of fractional derivative on the efficiency of these control strategies. Using a cost-effectiveness analysis method, our results show that combining awareness with diagnosis is the most effective strategy. To the best of our knowledge, this work is the first that propose a framework on the control of COVID-19 transmission based on a multi-age model with Caputo time-fractional derivative.

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## Sales Forecasting Using Recurrent Neural Networks

Manal Loukili, Fayçal Messaoudi, Mohammed El Ghazi,  
Raouya El Youbi

**Abstract** In this era of competitive business environment, customer satisfaction is a fundamental element. It is essential for organizations to have an efficient and accurate sales forecasting system, as more accurate sales forecasts allow for optimized resource utilization, scheduling, transportation, and effective inventory control. As a result, customer satisfaction is increased, and production costs are reduced. However, the volume of data is so enormous that humans alone are unable to process it manually. In order to achieve this, many machine learning and deep learning methods have been developed.

The objective of this article is to predict the next monthly sales of 10 stores. The data set used is available on the website "Kaggle", it contains data on the sales of 50 different items in 10 different stores over a period of 5 years [1]. For this we used a widely used recurrent neural network (RNN) algorithm called Long Short Term Memory (LSTM), and we evaluated the performance of the model on the basis of Mean Absolute Error (MAE), Root mean squared error (RMSE), and the coefficient of determination R-squared. As a result, the LSTM algorithm has proven to be performing well in predicting future sales, with a Root Mean Square Error of 19281.05, and a Mean Absolute Error equal to 16007.25, and a coefficient of determination of 98.68%.

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## Global dynamics of an age-structured viral infection model with general incidence function and humoral immunity

El Mehdi WARRAK<sup>1</sup>, Sara LASFAR<sup>1</sup>, Khalid HATTAF<sup>1,2</sup>, Noura YOUSFI<sup>1</sup>

### Abstract

During viral infections such as human immunodeficiency virus (HIV), lymphoma virus type I (HTLV-I) and hepatitis B virus (HBV), the production rate of viral particles and the death rate of infected cells depend on the infection age of cells. In addition, the age of infection plays an important role in the transmission of infectious diseases. On the other hand, the humoral immune response based on the antibodies which are produced by the B-cells and are programmed to neutralize the viruses.

Right now, numerous mathematical models possess been proposed and developed in order to know the dynamics of these viruses. Therefore, the main aim of this work is to propose an age-structured viral infection model with a general incidence function that takes into account the loss of viral particles due to their absorption into susceptible cells, in which the influence of humoral immunity and the infection age of the infected cells are considered. We first show that the model is mathematically and biologically well-posed. Further, the uniform persistence and the global dynamics of the model are investigated. The proposed model is described by ordinary differential equations (ODE) and partial differential equations. Also, two bifurcation parameters, the basic reproduction number  $R_0$  and the humoral immunity number  $R_1$  are derived.

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Analysis of a mathematical model for unemployment dynamics

**A.** Mohamed ELYAHYAOU, **B.** Saida AMINE

## Abstract

Unemployment is a major problem for all countries. In some poor countries, governments are unable to provide sufficient jobs for annual graduates as well as for the former unemployed, due to the lack of financial resources. Consequently, this causes the problem to persist and exaggerate. In this work, we improve the mathematical model that describe the interaction between unemployed people, employed people and the number of available vacancies introduced by the authors in [5]. Then we introduce the created vacancies jobs by means of logistic growth function. We investigate this problem using the theory of non-linear stability. Then we illustrate our theoretical finding by numerical simulations. Our qualitative results may be conductive to understand the dynamics behavior of unemployment phenomena.

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## Energy harvesting in a forced van der Pol oscillator using a delayed piezoelectric circuit.

Zakaria Ghouli

Polydisciplinary Faculty of Taroudant, University Ibn Zohr, Taroudant, Morocco

### Abstract

We investigate quasi-periodic vibration-based energy harvesting in a harmonically excited van der Pol oscillator coupled to a delayed piezoelectric coupling mechanism. We assume that the electrical component of the harvester is under a time delayed feedback such that the governing equation for the system can be written in the dimensionless form as

$$\ddot{x}(t) + x(t) - [\alpha - \beta x(t)^2]\dot{x}(t) - \chi v(t) = f \cos(\omega t) \quad (1)$$

$$\dot{v}(t) + \mu v(t) + \kappa \dot{x}(t) = \lambda v(t - \tau) \quad (2)$$

where  $x(t)$  is the relative displacement of the rigid mass  $m$ ,  $f$ ,  $\omega$  are, respectively, the amplitude and the frequency of the harmonic excitation,  $v(t)$  is the voltage across the load resistance,  $\alpha$  and  $\beta$  are the mechanical damping ratio,  $\chi$  is the piezoelectric coupling term in the mechanical attachment,  $\kappa$  is the piezoelectric coupling term in the electrical circuit,  $\mu$  is the reciprocal of the time constant of the electrical circuit,  $\lambda$  and  $\tau$ , are respectively, the feedback gain and the time delay in the electric circuit. The case of primary resonance, for which the frequency of the harmonic excitation is near the natural frequency of the oscillator, is considered. Analytical approximation of the quasi-periodic response and the corresponding power output are obtained using the double-step multiple scales method. The effect of the time delay on the energy harvesting performance is studied. It is shown that for appropriate combination of time delay parameters, quasi-periodic vibration can be used to scavenge energy over a broadband of the excitation frequency away from the resonance with a better performance.

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## On the date of the epidemic peak

M. MEZIANE, A. MOUSSAOUI

### Abstract

Dans [2], Bacaër s'est intéressé à la date du pic épidémique (qui correspond à la date où le nombre de personnes infectées atteint son maximum) dans le modèle S-I-R de Kermack et McKendrick ([1], chapitre 18). Il montre que le pic épidémique se produit à un temps  $T$  tel que  $T \sim (\ln N)/(a-b)$ , où  $N$  est la taille de la population,  $a$  est le taux de contact et  $b$  est le taux de guérison, avec l'hypothèse  $a > b$ . Dans [3], les auteurs se sont intéressés à l'étude du modèle épidémique S-E-I-R, avec pour objectif de déterminer la date du pic. Les auteurs ont trouvé une borne inférieure pour la date du pic épidémique et ont conjecturé que celui-ci a lieu au temps  $T \sim (\ln N)/\alpha$ , où  $\alpha$  est la plus grande valeur propre du système linéarisé. Sur la base de ces travaux, Nous nous sommes intéressés à déterminer une borne inférieure pour la date du pic épidémique et le nombre final d'individus ayant contracté la maladie pour deux modèles épidémiologiques. Le premier modèle est un modèle épidémique de base non structuré S-E-I-R (Susceptible Exposed Infectious Removed) avec deux classes d'individus latents : le modèle S-E1-E2-I-R, et le second modèle incorpore une structure d'âge continue pour les individus dans la phase des latentes. Les simulations numériques pour les deux modèles montrent un bon accord entre les résultats analytiques et les solutions exactes.

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## Some results on the generalized Apostol-Kolodner differential equation of the second order

<sup>a</sup>Mustapha Rachidi, <sup>b</sup>Mohammed Mouniane

<sup>a</sup>Institute of Mathematics - INMA, Federal University of Mato Grosso do Sul -UFMS,  
Campo Grande, MS, 79070-900, Brazil.

<sup>b</sup>Laboratory of Analysis, Geometry and Applications (LAGA), Department of Mathematics,  
Faculty of Sciences, Ibn Tofail University, Kenitra, B.P. 133, Morocco.  
E-mail addresses: mustapha.rachidi@ufms.br, mohammed.mouniane@uit.ac.ma

### Abstract

Linear Matrix Differential Equations play a significant role in many fields of mathematics, and especially in the modeling process of a variety of applications in many physical and engineering phenomena, in the economy, and elsewhere.

Among the most simple and fundamental equations are the first-order linear matrix equations with time-invariant coefficients.

This work concerns the generalized Apostol-Kolodner differential equations of the second order. We study the Apostol-Kolodner differential equation of the second-order, where we exhibit the properties of its solutions in terms of the square root of the matrices. The main purpose of the second step is to investigate the properties and solutions of the generalized Apostol-Kolodner matrix differential equation.

Our approach is based on some matrix square root properties, the Fibonacci-Hörner decomposition of matrix powers, and its related dynamical solution.

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## Strong and exponential stabilization for a class of boundary control systems

Abdellah Lourini<sup>1</sup>, Mohamed El-Azzouzi<sup>2</sup>, Mohamed Laabissi<sup>2</sup>

<sup>1,2</sup> LIMA, Department of Mathematics, FS-UCD, BP 20, 24000, El Jadida-Morocco.

<sup>1</sup> louriniabdellah3@gmail.com

### Abstract

In this talk, the following abstract linear boundary control system

$$\begin{cases} \dot{x}(t) = A_m x(t), & t > 0 \\ Qx(t) = Bu(t), & t > 0 \\ x(0) = x_0 \end{cases}$$

is considered on three Banach spaces  $X$ ,  $\partial X$  and  $U$ , where  $A_m: D(A_m) \subset X \rightarrow X$  is a closed, densely defined linear operator,  $Q \in \mathcal{L}([D(A_m)], \partial X)$  a boundary linear operator, where  $[D(A_m)]$  is the subspace  $D(A_m)$  equipped with the graph norm and  $B \in \mathcal{L}(U, \partial X)$  a control operator.

By an abstract approach and with generic assumptions, we study the stabilization of the abstract linear boundary control systems. Then we first transform their stabilization problem into a stabilization problem of a standard *unbounded linear control system*. *These results are then applied* to study the exponential stability of the boundary control system by using the spectral decomposition, also the strong stabilization of a class of Riesz-spectral boundary control problems is discussed. Our approach is based on the feedback theory of standard infinite dimensional linear systems. The effectiveness of the results of this paper has been illustrated by studying the stability of a class of heat equations.

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## Optimal Control for Smoking Epidemic Model

I. SMOUNI, B. KHAJJI, A. LABZAI, M. BELAM

### Abstract

The aim of this work is to propose a continuous-time mathematical model to study the dynamics of a population affected by smoking. Thus, the population is divided into eight classes: Potential smokers (P), light smokers (M), heavy smokers (D), sick heavy smokers (S), heavy smokers who join public treatment centers ( $C_{pb}$ ), heavy smokers who join private treatment centers ( $C_{pv}$ ), sick smokers who go to hospitals for treatment (H), individuals who quit smoking permanently (R). We provide three optimal controls which represent awareness programs through media and education for the light smokers, efforts to encourage the heavy smokers to join smoking treatment centers, and psychological support with follow-up for the individuals who quit smoking.

We use Pontryagin's maximum principle to solve the system. Consequently, the Numerical simulation via MATLAB confirms the theoretical results.

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## Review on Constructing Lyapunov Function in Delayed Reaction Diffusion Systems

Fatiha NAJM, Radouane YAFIA, M. A. AZIZ ALAOUI, Ahmed AGHRICHE

### Abstract

Motivated by some biological and ecological problems given by a delayed reaction diffusion systems with Neumann boundary conditions; and knowing their associated Lyapunov functions for delayed ordinary differential equations, we outline a method for determining their Lyapunov functions to prove the global stability.

The method is based on adding integral terms to the original Lyapunov function for ordinary differential equations. The new approach is not general but it is applicable in a wide variety of delay reaction diffusion models with one discrete delay or more, distributed delays and combination of both of them.

Firstly, we prove the approach for delayed reaction diffusion systems with one non-vanishing component. Secondly, we extend this study to multi-delays reaction diffusion systems with one non-vanishing component. Finally, we generalize this result to a reaction diffusion systems with multiple delays and multiple non-vanishing components. The obtained results can be applied to reaction diffusion systems with discrete delays or distributed delays or a combination of both of them. In the next work, we will try to generalize this approach to more general reaction diffusion systems with delays and other boundary conditions.

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## Fractional Critical problem involving a changing sign data

A. Senhadji Asma, B. Nasri Yasmina, C. Boukarabila Youcef  
Oussama

### Abstract

In recent years a great attention has been devoted to fractional and non-local operators of elliptic type, see, for instance [3], [4] and the references therein.

The study of this kind of problems is motivated by its various applications for examples : physical phenomena like flames propagation and chemical reaction of liquids, probability, population dynamics etc, for more details we refer the readers to the papers [1], [5].

In this work, we study the existence and multiplicity of solutions for a nonlocal problem involving critical exponent. Our goal is to find positive solutions for this problem although there is a changing sign data.

The mathematical interest of our problem lies in the fact that we have a lack of compactness due to the presence of the critical exponent and changing sign data which influences on the existence of positive solutions.

The idea of this work is to use an intermediate result and the sub-super solution method to prove that our problem admits a minimal positive solution. Taking into account the fact that our problem has a variational structure, we obtain a second solution by finding the critical points of the energy functional.

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# DEDS 2022

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## Estimation of the polynomial decay rate for a class of delayed semilinear systems

A. Delbouh<sup>1</sup>, Y. Benslimane<sup>1</sup>, H. El Amri<sup>1</sup>

<sup>1</sup>Applied Mathematics and computer sciences Laboratory, E.N.S.,  
Hassan II University of Casablanca

### Abstract

This communication presents the problem of feedback stabilization for distributed semilinear systems with time delay  $r > 0$  described as follows:

$$\begin{cases} \frac{dy(t)}{dt} = Ay(t) + v(t)By(t-r), t \geq 0, \\ y(t) = \varphi(t), \quad , t \in [-r; 0], \end{cases} \quad (1)$$

where  $y(t)$  is the state on a Hilbert space  $H$  endowed with the inner product  $\langle \cdot, \cdot \rangle$  and its corresponding norm  $\| \cdot \|$ . In addition, the linear operator  $A: D(A) \subset H \rightarrow H$  (generally unbounded) generates a strongly continuous semigroup of contractions  $S(t)$  on  $H$ . If  $y \in C([-r, +\infty[, H)$  and  $t \geq 0$ , then  $y_t \in C_r$  is defined by  $y_t(\theta) = y(t + \theta)$  for all  $\theta \in [-r, 0]$ , where  $C_r = \mathcal{C}([-r, 0], H)$  denotes the Banach space of continuous functions defined from  $[-r, 0]$  into  $H$ , endowed with the supremum norm  $\|\psi\|_{C_r} = \sup_{\theta \in [-r, 0]} \|\psi(\theta)\|$  and  $\psi \in C_r$  is a given initial function, while  $B$  is a nonlinear operator from  $H$  into  $H$  such that  $B(0) = 0$  (so that 0 is an equilibrium point), whereas  $t \mapsto v(t)$  is a scalar function which represents the control.

The main objective of this work is to show the estimation of the polynomial decay rate for the system (1) by using the following feedback control:

$$v_{log}^r(t) = \rho \log \left( 1 - \frac{\langle By(t-r), y(t) \rangle}{1 + |\langle By(t-r), y(t) \rangle|} \right), t \geq 0, \rho > 0.$$

Both strong and weak stabilization for the system (1) are investigated under sufficient conditions and an illustrating example is given.

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## Sentiments Analysis Using RNN: Review

H. SAKHI<sup>1</sup>, S. EL FILALI<sup>2</sup>, H. BENLAHMAR<sup>3</sup>, S. OUAHABI<sup>4</sup>

<sup>1,2,3,4</sup>, *LTIM, Hassan II University of Casablanca, Morocco*

### Abstract:

Sentiment analysis or opinion mining is the set of operations aiming at extracting emotions from a set of data, and classifying them afterwards according to classes, positive or negative as a first approach, we can improve our results adding other classes: positive, negative, neutral ....

Sentiment analysis has been implemented in various fields such as politics, business and through it we will be able to make major decisions.

Machine learning algorithms dealing with classification cases have proven their efficiency in the treatment of different sentiment analysis problems.

In this project, we want to highlight a classical deep learning model, recurrent neural networks (RNN) and their ability to handle similar cases: comment classification positive-negative (comments scrapped through twitter).

This paper will focus on the architecture of the network, the parameters, the analysis including the steps to follow to build it as well as the various other applied techniques recommended in the same direction.

We will also discuss some intermediate approaches and then compare them with the advantages and limitations of each.

We will also discuss some challenges related to the Overfitting, and propose improvements in this direction.

Finally, we will discuss some proposed directions to improve the results obtained.

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## Digraph Characterization of Null Controllability for Switched Positive and Periodic Positive Systems

A. Ilyas Sitli, B. Fouad Lahmidi, C. Abdelwahed Namir

### Abstract

In this paper, we deal with positive discrete-time linear control systems in the state-space model, i.e., systems whose states and inputs are nonnegative. This work is based on the characterization of the null controllability in discrete-time positive linear systems. A positive system is controllable if it is possible to transfer it from an arbitrary non-negative initial state to a zero state using only certain admissible non-negative controls. We solve this problem with a technique based on graph theory. In this paper, we introduce the characterization of the null controllability property in digraph form for two classes of discrete-time positive linear systems. The first one is switched systems, which are a type of hybrid dynamical system and have piqued the interest of engineers and applied mathematics in recent years. A switched system is made up of a finite number of differential or difference subsystems and a switching law that determines which subsystem is active at any given time. The second one is the periodic system. The study of periodic systems is motivated by the fact that many practical systems possess periodic characteristics. Some examples are given to verify the theoretical results.

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## Modèle multi-physique de dynamique des populations Meryem Bensenane

### Abstract

Dans ce exposé, un modèle à temps continu est utilisé pour étudier les effets des aires marines protégées et de leurs tailles sur les captures, les aires marines protégées notées par MPA sont des zones dans la mer où la pêche est interdite afin de laisser les poissons se reproduire. Le modèle qu'on propose comprend deux échelles de temps: une échelle rapide associée aux mouvements rapides des poissons entre les sites et une échelle lente correspondant à la la croissance de la population et la dynamique de l'effort de pêche. L'existence de deux échelles de temps permet d'utiliser la méthode d'agrégation de variables pour construire un modèle réduit décrivant la dynamique du stock total de poissons et de l'effort de pêche. Ce modèle agrégé décrit la dynamique à long terme, l'objectif est d'étudier ce modèle pour déterminer les effets de la création des MPA sur le stock de poisson et sur les captures à long terme, en étudiant ce modèle agrégé, nous montrons l'existence d'une taille optimale de la réserve marine qui maximise la capture totale de poissons à l'équilibre. Les résultats des simulations suggèrent que la création d'une réserve marine protégée conduira toujours à une augmentation de la biomasse totale de poissons, une taille optimale d'une réserve marine permet de maximiser la capture à l'équilibre.

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## Exponential stabilization of distributed bilinear time-delay systems of neutral type

M. ERRAKI, A. EL HOUGH, A. ATTIOUI

### Abstract

The stabilisation problem of retarded bilinear systems in infinite dimensional has been studied in many works. In [1], the exponential stabilization issue for distributed bilinear systems without delay was obtained, the same result was obtained in [3] for a class of bilinear systems with time delay. Furthermore, the authors in [5] have considered the problem of feedback stabilisation of bilinear systems with discrete delay in a Hilbert space.

On the other hand, there have been a number of studies on stabilization of neutral systems in finite dimensional space by many researchers, using different techniques [2,4].

Motivated by the above discussion, we consider, in this paper, the exponential stabilization for a class of distributed bilinear time delay systems of neutral type, evolving in a Hilbert state space .To achieve this, we propose a continuous and bounded feedback control. Sufficient conditions in term of observation estimates are given. Moreover, we consider the decomposition of the state space via the spectral proprieties of the systems to discuss the stabilization issue. In this case, the stabilization of a such system reduces stabilizing only its projection on a suitable subspace under a weaker observability assumption. Applications to wave and heat equations with numerical simulations are considered to illustrate the effectiveness of the obtained theoretical results.

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## Impact of wind speed on optimal fishing effort

A. Labriji, Y. El Foutayeni, M. Rachik

### Abstract

Following the different developments seen in the literature on the population dynamics of fish impacted by fishing activity (see [2, 3, 5, 6] for example), we seek to highlight the impact of wind speed on the fishing effort and, consequently, on the catches, the profit, and also the biomass of the species caught. This is to show the importance of the wind factor during fishing. In the literature on bio-economic models, Agmour et al. (2020) [1] succeeded in introducing meteorological factors during the formalization of mathematical models to obtain results closer to reality, which highlighted the impact of the wind speed on the fishing effort, the catch, and the profit of the fishers. We rely on this work and propose an extension by adding a probabilistic dimension to model wind fluctuations during the different months of the year using Markov chains while considering a single marine fish species, the *Sardina pilchardus*. This small silvery pelagic is well known due to its historical abundance on the Moroccan coasts and is one of the most exploited species in Morocco. To this end, we consider a bioeconomic model for *Sardina pilchardus*, as well as a probabilistic model to capture weather changes in southern Morocco, one of the windiest coastal regions of the country. These two models allow us to calculate the optimal daily fishing effort for a whole year under weather constraints, and allow us to respect the sustainability of the stock of the marine species studied [4]. We can then deduce the catch, profit, and biomass status from the calculated effort. We then compare these results to those of a deterministic model to show the effect of the weather factor on the fishery.

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## Étude de la bifurcation de Bogdanov-Takens d'un système couplé de FitzHugh–Nagumo à retards

A. Achouri Housseem, B. Aouiti Chaouki, C. Ben Hamed Bassem

### Abstract

Dans cette présentation, on fait l'étude d'une bifurcation de Co dimension deux; la bifurcation de Bogdanov-Takens, d'un système FitzHugh–Nagumo couplé avec retards. En fait, on peut voir cette étude comme étant une étude de la sensibilité du comportement de ce système par ces paramètres.

Dans une première étape, le point critique auquel une racine nulle de multiplicité deux apparaît dans l'équation caractéristique est construit.

Dans la deuxième étape, afin de s'assurer que toutes les racines de l'équation caractéristique à l'exception de la racine du double zéro ont des parties réelles négatives, on étudie les zéros d'un polynôme exponentiel du troisième et du quatrième degré. Par conséquent, les valeurs critiques où se produit la bifurcation Bogdanov-Takens sont dérivées.

Dans la troisième étape, en utilisant la théorie de la forme normale et la réduction sur la variété centrale correspondante à l'espace propre généralisé associé à la valeur propre zéro, la forme normale réduite de dimension deux est obtenue et ses comportements dynamiques sont étudiés. Finalement, un exemple numérique est donné pour démontrer nos résultats.

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## Optimal Control of Ship Collision Avoidance Problem Yousra MELHAOUI, Khalifa MANSOURI, and Mostafa RACHIK

**Abstract** The continuous increase of maritime traffic amplified the severity of the collision risk issue in the maritime domain. Therefore, the calculus and optimization of ship navigation without collision risks have been known as a major challenge for the scientific research community. Several solutions were proposed to enhance maritime safety. The topic was covered as an optimal control problem with state constraints using Nonlinear Model Predictive Control in order to consider the nonlinearity of the ship's motion. Other researches relied on calculation risks of collisions in ocean navigation by metaheuristic methods or by neural networks in order to cover multi-ship collision risk situations. In this work, a detailed description of necessary elements used in the analysis of the maritime navigation without collision issues is presented including the ship motion, the International Regulations for Preventing Collisions at Sea COLREGs rules, and the navigation cost. This work provides a literature contribution on applying optimal control techniques to maritime safety. By including ship motion in our study, we present a realistic controlled model that represents real-time ship navigation. By the mean of Pontryagin's maximum principle, we develop a control strategy that provides to the ship autopilot the rudder and the propulsion values should be applied as a system control in real-time navigation. Simulation results obtained, indicate that the control strategy helps in avoiding ship collision risk simultaneously and effectively with respect to COLREG rules and cost minimization.

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## New Comments on Qualitative Analysis of Solutions of Integro-Differential Equations

Osman Tunç

### Abstract

In this work, we deal with some properties of solutions to an integro-differential equation by Lyapunov-Krasovkii functional method. We prove some new results on qualitative behaviors of solutions of considered equation. An example is given to illustrate the application of the results of this paper.

**Keywords:** Integrability, stability, instability, Lyapunov-Krasovkii functional

**Subject Classification:** 34D05, 34K20, 45J05

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## Modeling and teaching the dynamics of hepatitis B virus in Moroccan education

Nabila BEQQALI<sup>1,2</sup>, Khalid HATTAF<sup>1,2</sup>, Naceur ACHTAICH<sup>1</sup>

### Abstract

Hepatitis B virus infection (HBV) is a major global health problem that can cause acute or chronic infection and puts people at high risk of death from cirrhosis and liver cancer. For instance, hepatitis B caused 887000 deaths in 2015, mostly from complications (including cirrhosis and hepatocellular carcinoma) according to World Health Organization (WHO). For these reasons, many mathematical models have been proposed and developed to better understand the dynamics of HBV infection. However, there are a few works that focus on teaching and learning these models. Therefore, the main objective of this work is to model and teach the dynamics of HBV in high school mathematics education. To achieve this objective, we first present the teaching of infectious diseases and more precisely the disease of hepatitis B in the Moroccan curriculum. In addition, we develop various activities to build the visual, discrete, and continuous version of an HBV epidemic model. These activities can develop several skills and abilities such as scientific reasoning, problem solving, and communication skills. Furthermore, the construction of this HBV epidemic model will be done through the ordinary differential equations (ODEs) which are programmed in the teaching of mathematics in high Moroccan school.

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## ASYMPTOTIC BEHAVIOR OF POSITIVE SOLUTION OF SEMILINEAR FRACTIONAL ELLIPTIC PROBLEM

A.A.BATAHRI, A.ATTAR

### Abstract

The aim of this work is to generalize the results obtained in [2] by Boccardo et al. in the nonlocal case.

Let consider the following semilinear fractional problem with increasing power term

$$(P_{\lambda,m}) \begin{cases} (-\Delta)^s u + u^{m-1} = \lambda u^{p-1} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases}$$

Where  $s \in (0,1)$ ,  $\Omega$  is a bounded domain in  $\mathbb{R}^N$ ,  $p \in [2, 2_s^*)$ , the real parameter  $\lambda > 0$  and  $m$  sufficiently large such that  $2 < p \leq 2_s^* < m$ .

The operator  $(-\Delta)^s$  is the Fractional Laplacian given by

$$(-\Delta)^s u(x) := a_{N,s} \text{ P.V. } \int_{\mathbb{R}^N} \frac{u(x) - u(y)}{|x - y|^{N+2s}} dy, \quad s \in (0, 1),$$

When  $s=1$ , Boccardo et al. [2] studied the local sub-case of our problem. Where they proved the existence of sequence of solutions and an  $L^\infty$  priori bound which plays a crucial role to study the asymptotic behavior of the positive solutions as  $m$  tends to  $\infty$ .

Motivated by [2], in this work we prove the existence of two positive sequences of solutions exploiting variational methods to the above problem for  $m$  large but fixed.

Therefore, we prove a regularity results to be able to perform our asymptotic analysis, and we will show that the behavior of sequences of solutions is determined by a limit problem as  $m$  tends to  $\infty$ .

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## Stability analysis of Singular two dimensional systems

K. Benyettou<sup>1</sup>, D. Bouagada<sup>2</sup>, M. A. Ghezzar<sup>3</sup>

**Abstract** The development of control theory over the past decades has given us many singular models described by the Roesser model that have many applications such as RLC circuit problems, chemical reaction, Robotics, signal and digital image processing [1,2,3,4,5,6].

The most essential and fundamental problem for the design and analysis of systems is the stability test. The notion of stability of the Roesser model was implemented By Kaczorek et al. In [4]. In the literature, many authors have been introduced some approach to test the stability of these class of systems, for example Bouagada et al. In [2] defined a new LMI criteria as well as an asymptotic stability test for a two-dimensional linear systems for the class of singular Fornasini-Marchesini models. Ghezzar et al. in [3] has developed results extended to the general 2D discrete-time and continuous-discrete-time singular Lyapunov systems and presented a sufficient conditions for asymptotic stability test in terms of linear matrix inequalities (LMI's). Another LMI method was investigated for the stability of 2D state-space singular models by Elosmani et al. [1] who analyzed the stability of multidimensional systems and presented new sufficient conditions for asymptotic stability in terms of linear matrix inequalities.

The aim of this work is to present a contribution to test the stability of two dimensional singular linear systems described by the Roesser model, and to provide sufficient conditions for asymptotic stability tests in the context of linear matrix inequalities (LMIs) and polynomial tests as an extension based on [2, 3, 4].

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## Exponential stability of retarded recurrent neural networks

S. Othmani, N.-e. Tatar

**Abstract** Artificial intelligence is the root of artificial neural networks, which have numerous applications in various fields such as biology, economics, finance and many areas of engineering. In extension of the familiar Hopfield neural networks, Kosko first presented a new class of recurrent networks in 1987 named bidirectional associative memory (BAM) neural networks [2]. Their conception embraces two interconnected hidden neural layers where the neurons of each layer are not linked together. Such networks can be successfully applied in different fields as pattern recognition, signal and image processing, automatic control, associative memory, and so on. In the hardware implementations, time delays occur owing to the finite switching speed of the amplifiers when the neurons communicate with each other. These can lead to oscillations, divergences and instabilities, that have detrimental effects on systems. Neural networks generally have a spatial span caused by the multiplicity of parallel paths with axons of different size and length, and therefore a distribution of propagation delay is produced throughout a time interval. Furthermore, activation functions, which relate the inputs to the outputs of neural networks, are a core element of artificial neural networks. Usually, these functions of hidden neurons have degree of nonlinearities that is significant in the majority of applications of artificial neural networks. At the beginning of the study of neural networks, such functions were assumed bounded, smooth and monotonic [1, 5]. Subsequently, a slight relaxation of these conditions to Lipschitz type took place, which has been widely used in the literature [3, 4]. Since non-Lipschitz activation functions are important in implementations, it is necessary to relax the common Lipschitz condition. In this talk, by means of a more relaxed condition on the activations, sufficient conditions to ensure exponential stability for BAM neural networks with distributed delays are derived. The argument is based on a nonlinear Halanay inequality as well as some analytical techniques. The accuracy of the theoretical results is validated by a numerical example.

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## New results on impulsive type inertial bidirectional associative memory neural networks

Chaouki Aouiti, Mahjouba Ben Rezeg

**Abstract** This paper is concerned with inertial bidirectional associative memory neural networks with mixed delays and impulsive effects. Note that Bi-directional Associative Memory Neural Networks (BAMNNs) was investigated by kosko in 1982. This kind of neural network has been proved to have widespread applications in various fields such medical image edge detection, medical event detection in electronic health records, diagnosis prediction in health care, pattern recognition and robotics. For this reason, there have been extensive results on the problem of dynamic analysis of BAMs [1]. Researchers have also investigated neural network's (NNs) by adding an inertial term. This model was first introduced by Wheeler and Schieve [2]. Recently, inertial neural network's with a delay have been widely investigated by many authors because of their role in generating complicated bifurcation behavior and chaos. The mathematical modeling of various physical processes gives rise to anti-periodic solutions. Okochi studied the first anti-periodic solutions for nonlinear evolution equations. The investigation of anti-periodic solutions is an important subject because of its applications in engineering, physics, control theory. In NN theory, much attention has been paid to the study of anti-periodic oscillations of different types of neural networks [3,4]. Numerous biological systems such as biological neural networks and bursting rhythm models in pathology, as well as optimal control models in economics, frequency-modulated signal processing systems, and flying object motions, are susceptible to immediate problems and experience unexpected changes. The term of these progressions is exceptionally short and irrelevant in relation to the duration of the process considered and can be thought of as momentary changes or impulses. In neural network theory, systems with short-term perturbations are naturally described by impulsive differential equations [4]. In this study, we establish new results concerning the existence, uniqueness, and global exponential stability of anti-periodic IBAMNN solutions with mixed delays and impulsive effects. New and practical conditions are given to study the existence, uniqueness, and global exponential stability of anti-periodic solutions for the suggested system. We use differential inequality techniques to prove our main results. Finally, we give an illustrative example to demonstrate the effectiveness of our new results.

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## Contrôle et analyse de stabilité de l'épidémie d'obésité.

A.ELMANSOURI, B.KHAJJI, A. LABZAI, M. BELAM

L'obésité et le surpoids se caractérisent par une accumulation anormale, ou excessive, de tissus adipeux (contenant de la graisse) pouvant représenter un danger pour la santé. **L'obésité** dégrade considérablement la qualité de vie d'un individu adulte et peut avoir de graves conséquences sur sa santé.

L'objectif de ce travail est de proposer un modèle mathématique pour étudier la dynamique comportementale d'une population touchée par la maladie de l'obésité.

Ainsi, la population étudiée est divisée en six compartiments : sensible (S), exposé (E), légèrement obèse (I1), modérément obèse (I2), très obèse (I3) et guéri (R). Notre étude se pose sur deux axes principaux :

- Proposer quatre contrôles : la sensibilisation par l'éducation et les médias, les programmes alimentaires et sportifs, le traitement médical avec des médicaments et le traitement avec une intervention chirurgicale, et caractériser les contrôles optimaux à l'aide du principe de maximum de Pontryagin.

- Utiliser les critères de Routh-Hurwitz dans l'objectif de construire les fonctions de Lyapunov, et discuter la stabilité locale et globale.

Enfin, nous cherchons à confirmer l'analyse théorique par des simulations numériques via Matlab.

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## Conformal Fractional Cauchy Problem with a Measure of Non compactness in Banach spaces

K. HILAL, A. KAJOUNI , N.CHEFNAJ

### Abstract

The differential equation with the conformable fractional derivative is introduced by T.Abdjawad[2].And A.Kajouni, M.Chafiki, K.Hilal, M.Oukessou gives new form of this derivative and some application [1]. This new fractional derivative quickly becomes the subject of many contributions in several areas.

This last derivative is generalization of the classical derivative because This novel fractional derivative respects the most properties of the classical derivative ,an important tool in the modeling of phenomena in several scientific fields such as physics ,engineering ,control theory ....

In this work, we prove the existence of mild solution of conformable fractional differential equation with a measure non compactness in Banach spaces , precisely the Hausdorff measure of noncompactness .for the more about a measure non compactness, we refer to [3] . The main results are based on semi group theory combided with Darbo-Sadovskii fixed point theorem.

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## on fractional model in magneto-viscoelastic interactions

M. El Idrissi, E-H. Essoufi, C. Ayouch

### Abstract

The magnetic and elastic properties of ferromagnetic materials and others depend on each other. The different couplings between these properties are called magnetoelastic effects. These effects can be separated into two main categories, direct effects and their inverse effects. The most important direct effect is magnetostriction. The latter reflects the phenomenon whereby a ferromagnetic sample deforms due to magnetic interactions that may be either within the sample itself (spontaneous magnetostriction) or a consequence of an external magnetic field (forced magnetostriction). This magnetostriction results in a state of constraint (even in the absence of any external stress) which is responsible for the rearrangement of the domains, as well as other phenomena. In the magnetoelastic model, the magnetization results from non-mechanical external forces and is not influenced by the mechanical state of the material. In this paper, we propose a model for the theoretical study of the interaction between the elastic and magnetic processes, it combines phenomenological constitutive equations for the magnetization  $m$  and the displacement  $u$ . The nonlinear parabolic hyperbolic coupled system describing the dynamics in  $Q = (0, T) \times D$  ( $D$  is a bounded open set of  $R_d, d > 1$  and  $\partial D$  its boundary) is given by

$$\begin{cases} m_t = \vartheta m \times H_{eff} - \mu m \times (m \times H_{eff}) \\ \rho u_{tt} - \operatorname{div} \left( S(u) + \frac{1}{2} \mathcal{L}(m) \right) = 0 \end{cases}$$

The first equation (1), well known in the literature, is the Landau-Lifshitz equation and the second equation (2) describes the evolution of the displacement.

We use the Faedo Galerkin/Penalty approach to verify global existence.

To prove the convergence of nonlinear terms, some commutator estimations are used.

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## An age-structured insect population model with diapausal stage, with a contrability study.

S.M.Abderrahim, B. Ainseba, A. Moussaoui

### Abstract

Pests insect are causing major damage to agriculture production around the world. Over the years, many different practices have developed to manage the pest populations and to limit the losses caused by the various species. Sterile insect, mating disruption, and mass annihilation techniques are examples of methods that are part of integrated control strategies for pests [4], using chemical pesticides favours the development of resistances in the pests, which create mechanisms to resist these pesticides.

In the literature, many mathematical models have been developed to explain the pest population management process, in [1], authors have formulated a Mathematical Model that describes the population dynamics of the life cycle of grapevine insect pests that takes into account climate and grape varieties, showed the existence and uniqueness of solutions. The optimal control of the model was studied in [2]. The Authors in [3] are interested in the existence and the uniqueness of a nonnegative solution for a four-step structured model with second derivative. In [1, 3], the authors divided the life cycle of the pest into 3 stages: eggs, larvae and adults. As we have seen, most insects build cocoons to protect the pupa from adverse conditions and predators. For that reason, we are very interested in looking at a mathematical model that describes the life cycle of insect pests that incorporates the cocoon and pupal stages. And in this presentation, we show a dynamic model of pests with cocoon stage, the objective being to study the existence and the unicity of solutions as well as the optimal control of the pest by elimination of the population of eggs. Illustrations of numerical simulations are given by using parameters and functions that are estimated in [5].

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## LIST OF PARTICIPANTS

Family Name	First Name	University	Country
ABBES	Kadyri	Chouaib doukkali university	Morocco
ABDALLAH	Roubi	University of Mohamed Khider	Algeria
ABDELFATTAH	El achab	University Cadi Ayyad	Morocco
ABDERRAHIM	Sidi mohammed	University of Tlemcen	Algeria
ABID	Soufiane	HASSAN 1er	Maroc
ACHOURI	Houssem	University of Carthage	Tunisia
AFOUKAL	Abdallah	Sultan Moulay Slimane University	Morocco
AGHRICHE	Ahmed	Sultan Moulay Slimane University	Morocco
AIT OUSSAID	Nouhayla	Ibn Zoher University	Maroc
ALAA	Nour eddine	Cadi Ayyad	Morocco
ALAOUI	Badreddine	EHTP	Morocco
ALLAL	Brahim	Faculty of Applied Sciences	Morocco
AMEUR	Meryem	Sultan Moulay Slimane	Morocco
AOUITI	Chaouki	Carthage	Tunisia
AQALMOUN	Mohamed	Sidi Mohamed Ben Abdellah University	Morocco
AQEL	Fatima	University Hassan I	Morocco
ASSALI	El abed	University of carthage	Tunisia
ASSILA	Chaimaa	Mohammed V university	Morocco
ATMANI	Somia	University of Abou Bakr Belkaïd	Algeria
AYACHI	Moez	University of Gabes	Tunisia
AZRAIBI	Ouidad	Sidi mohammed ben abdellah	Morocco
BACHMAR	Aziza	University of Setif1	Algeria
BADR	Sanaa	University Hassan II of Casablanca	Morocco
BATAHRI	Asma amira	University of Tlemcen	Algeria
BATAHRI	Asma amira	Aboubakr Belkaid	Algeria



BAYMOUT	Louiza	University of Bordj Bou Arreridj	Algeria
BEDJGUELEL	Chabane	Abderrahmane Mira	Algeria
BELFAR	Ahlam	University of Bordj Bou Arreridjr	Algeria
BEN AHMED	Imen	Université de Carthage	Tunisie
BEN LAHBIB	Abdelaziz	University Mohammed V	Morocco
BEN REZEK	Mahjouba	University of Carthage	Tunisia
BENADDY	Abdellah	Hassan first university of settat	Morocco
BENAMARA	Ibtissam	Mohammed V University	Morocco
BENMERROUS	Abdelmjid	Sultan Moulay Slimane University.	Morocco
BENSENANE	Meryem	Tlemcen	Algeria
BENTACHFINE	Ilyass	FST Beni Mellal	Morocco
BENYETTOU	Kamel	Abdelhamid ibn badis university	Algeria
BEQQALI	Nabila	Hassan II University of Casablanca	Morocco
BESSIFI	Maissa	Bizerte Faculty of Science	Tunisie
BOUAJAJI	Rachid	Hassan II	Morocco
BOUALLALA	Mustapha	Cadi Ayyad	Morocco
BOUAMAIED	Ghada	Université de Carthage	Tunisie
BOUCHRA	Lahmidani	Sultan Moulay Sliman	Morocco
BOUKANJIME	Brahim	Ibn Tofail University	Morocco
BOUKARABILA	Siham	University of Tlemcen	Algeria
BOULAHROUZ	Tarik	Université Mohammed Premier Oujda	Maroc
BOUMASMOUD	Soufiane	Cadi Ayyad University	Maroc
BOUMAZOURH	Athmane	Sidi Mohamed Ben Abdellah University	Morocco
BOUZIANE	Soukaina	Faculty of sciences ben ms'ik	Morocco
BRIKI	Imane	Mohammadia school of engineering	Morocco
CHAHRA	Kechar	University of Souk Ahras	Souk Ahras
CHANNAN	Khadija	Sultan Moulay Slimane	Morocco
CHEFNAJ	Najat	Soltan Moulay Slimane University	Morocco

CHEMS EDDINE	Nabil	Mohammed V University	Morocco
DAHI	Ibrahim	University Moulay Ismail	Morocco
DAIRA	Ibtissem	Souk Ahras University	Algeria
DEKKAR	Imane	Mohammed Seddik Ben Yahia	Algeria
DELBOUH	Ahmed	Hassan II University	Morocco
DJERADI	Fatima siham	Amar Telidji University of Laghouat	Algeria
DRIDI	Farah	Higher Institute of Management	Tunisia
EDDAMANE	Elhousseine	FS MEKNES	Morocco
EL ATTAOUY	Meryem	Cadi Ayyad University	Morocco
EL BAZ	Oumaima	Faculté des sciences Ben M'sik	Morocco
EL FOUTAYENI	Youssef	Hassan II University of Casablanca	Morocco
EL GHABI	Malika	Faculty of science and technology	Morocco
EL HAMMANI	Mohammed	University of Mohammed V	Maroc
EL HARMOUCHI	Nour-eddine	University Hassan II of Casablanca	Morocco
EL IDRISSE	Mourad	Abdelmalek Essaâdi University	Morocco
EL IDRISSE	Mohamed	University Hassan I	Morocco
EL JIANI	Laila	Hassan II University	Morocco
EL KARIMI	Mly ismail	Hassan II University of Casablanca	Morocco
EL KARIMI	Mly ismail	Haasan II	Morocco
EL KOUFI	Amine	Hassan II- casablanca	Morocco
EL OTMANI	Fadwa	Hassan 2 of casablanca	Morocco
EL OUAARABI	Mohamed	Sultan Moulay Slimane University	Morocco
EL OUARDY	Ilham	University moulay ismail	Morocco
EL YOUNOUSSI	Majda	Hassan 2 university	Morocco
ELAASSRI	Abdelwahab	Mohamed Premier	Morocco
ELGHANDOURI	Mohammed	Cady Ayyad University	Morocco
ELHAFIANE	Zakaria	Cady Ayyad university	Morocco
ELHALOUI	Loubna	Faculty of Sciences Ben M'Sik, Hassan	Morocco

ELHIA	Mohamed	Hassan II University of Casablanca	Morocco
ELKARMOUCHI	Maha	Faculty of sciences Ben M'Sik	Morocco
ELKHAIAR	Soufiane	Ibn zohr university	Morocco
ELMADANI	Youssef	Mohammed V University in Rabat	Morocco
ELMADDAH	Youness	Université Hassan II de Casablanca	Maroc
ELMANSOURI	Abdelbar	Sultan Moulay Slimane University	Morocco
ELYAHYAOUI	Mohamed	Hassan II University of Casablanca	Morocco
EL-YAHYAOUI	Houria	Sidi Mohamed Ben Abdellah University	Morocco
ERRAKI	Mohamed	University of Hassan 2	Morocco
ES-SAIYDY	Mohssine	Moulay Ismail University	Maroc
ESSOUNAINI	Abdelhak	University Hassan II of Casablanca	Morocco
ETTAYB	Jawad	Sidi Mohamed Ben Abdellah	Morocco
FAKHRI	Marwan	Hassan 2 university	Morocco
FATIHA	Mesdoui	Jijel university	Algeria
GHANMI	Boulbaba	Gafsa	Tunisia
GHENIMI	Seyf eddine	Mohamed Boudiaf	Algeria
GHOULI	Zakaria	Polydisciplinary Faculty of Taroudant	Maroc
HADDANI	Ismail	Moulay Ismail	Morocco
HAJJI	Youssef	Abdelmalek Essaadi	Morocco
HANNABOU	Mohamed	University Sultan Moulay Slimane	Morocco
HARDOUL	Asmae	Ibn Tofail	Morocco
HATHOUT	Fatima zohra	Aboubekr Bekaid	Algeria
HATIM	Khalid	Hassan II University	Morocco
HELAL	Mohamed	Univesity of Djillali Liabes	Algeria
HOURRI	Maryem	Cadi Ayad University	Morocco
ID OUAZIZ	Saida	Faculcte of science and technology	Morocco
IMANE	Dekkar	Mohammed Seddik Ben Yahia	Algeria
ISMAIL	Oumadane	Mouly ismail meknes	Maroc

JALLOULI	Hediene	University of Carthage	Tunisia
JAOUHARI	Mounir	HASSAN II	Morocco
JAWAD	Ettayb	Usmba, Fsdm, Fez	Morocco
KADA	Driss	Faculty of Sciences Ben M'sik	Maroc
KEDDAR	Naima	Higher Scholl of Management	Algeria
KHAJJI	Bouchaib	Faculté des sciences Ben M'sick	Maroc
KICHA	Abir	University of Mohamed Seddik	Algeria
KOURRAD	Ahmed	Hassan II University of Casablanca	Morocco
LABRIJI	Ali	Hassan 2 University	Maroc
LAGOITI	Abdelilah	Faculté des sciences ben m'sik	Maroc
LAHYANI	Ahmed	Soultan Moulay Sliman	Maroc
LAKHDI	Abdessamad	University Moulay Ismail	Morocco
LASFAR	Sara	Hassan II university of casablanca	Morocco
LOUKILI	Manal	Sidi Mohamed Ben Abdellah University	Morocco
LOURINI	Abdellah	Chouaib Doukkali University	Morocco
MAADAN	Hicham	Sultan Moulay Slimane University	Maroc
MAKRAN	Noreddine	Mohammed Premier	Morocco
MEBARKA	Sattaf	Faculty of Exact Sciences	Algeria
MEHDAOUI	Mohamed	Moulay Ismail University	Morocco
MELHAOUI	Yousra	Hassan II University Casablanca	Morocco
MELLAH	Zhor	Mohammed Premier	Morocco
MESDOUI	Fatiha	Jijel university	Algeria
MESSAOUDI	Fayçal	Sidi Mohamed Ben Abdellah University	Morocco
MESSAOUDI	Faycal	Sidi Mohamed Ben Abdellah	Morocco
MEZIANE	Mohammed	Abou bekr BELKAID Tlemcen	Algeria
MOHAMED	Hammoumi	Sidi Mohamed Ben Abdellah University	Morocco
MOHAMED CHAOUKI	Tarchi	Université de Carthage	Tunis
MOHSINE	Hakima	Mohammed First	Morocco

MOUMNI	Alhabib	Moulay Ismail	Morocco
MOUNIANE	Mohammed	Ibn Tofail University	Morocco
MUSTAFA	Irfan	Allama Iqbal Open University	Pakistan
NACERI	Mokhtar	ENS of Laghouat	Algeria
NAJM	Fatiha	Ibn Tofail	Maroc
NIYONKURU	Venant	Kenyatta University	Kenya
NOHAIR	Lotfi	FSBM	Morocco
NOUREDDINE	Elharrar	FS Eljadida	Morocco
OTHMANI	Sakina	University of Science and Technology Houari Boumediene	Algeria
OUBOUISK	Mhamed	Faculté des Sciences et Techniques	Maroc
OUELD DRIS	Ilham	Cadi Ayyad University	Morocco
OULTOU	Abderrahmane	SULTAN MOULAY SLIMANE	Morocco
OUSTANI	Salah eddine	Ibn Tofail University	Morocco
RACHID	Kamal	Ibn Tofial	Morocco
RIDA	Oussama	Hassan First	Morocco
ROSSAFI	Mohamed	University Sidi Mohamed Ben Abdellah	Morocco
SABRI	Abdelali	Université Hassan Premier	Morocco
SAKHI	Hasnae	Faculty of science Ben M'Sik -FSBM	Morocco
SAKKOUM	Ayou	University Hassan II	Morocco
SENHADJI	Asma	University of Abou Bakr Belkaid	Algeria
SITLI	Ilyas	University Hassan II	Maroc
SMOUNI	Imane	Universite sultan moulay slimane	Morocco
SRATI	Mohammed	Sidi Mohammed Ben Abdellah	Maroc
TAHRI	Kamel	High School of Management	Algeria
TALHA	Adnane	Sidi mohammed ben abdellah	Morocco
TAYBI	Mohamed amine	Faculty of Sciences Dhar El Mahraz	Morocco
TOGOU	Abdelkarim	Abdelmalek essaadi	Morocco
TOUATI	Farid	University of Carthage	Tunisia

TOUKMATI	Ahmed	University of Abdelmalek Essaadi	Morocco
TUNÇ	Cemil	Van Yuzuncu University	Turkey
TUNÇ	Osman	Van Yuzuncu Yil University	Turkey
WARRAK	El mehdi	Hassan II	Morocco
YOUNES	Abdelbadie	University of Abou Bakr Belkaïd	Algeria
YOUSFI	Noura	Hassan II de Casablanca	Maroc
YÜKSEKKAYA	Hazal	Dicle University	Türkiye
ZAKRAD	Az-eddine	Cadi ayyad university	Morocco
ZARHOUNI	Mohammed	Moulay Ismail University	Morocco
ZARROUK	Mohamed mustapha	University Moulay Ismail	Morocco
ZERBIB	Samira	Sultan Moulay Slimane University	Morocco
ZIZI	Zakaria	Cadi Ayyad university	Morocco
ZOUHAIR	Walid	Cadi Ayyad	Morocco
ZRAR	Mustapha	University Hassan II of Casablanca	Morocco